OPTIMAL DISTRIBUTION OF FINANCIAL RESOURCES IN PUBLIC MULTI-PROJECT MANAGEMENT

Ratko Spaic

STARK, ltd. Ljubljana, Slovenia

Short Abstract:
Planning of public infrastructure building is a very complex management task, which belongs to the concept of multi-project management. In the multi-project management there is a difficulty of adjusting the activities in a number of projects and making optimal distribution of financial resources. This paper analyses a practical model of distribution of limited budget resources in an investment program for urban public infrastructure.

Keywords: planning, optimization, multi-project management, public infrastructure.

The actual needs for public objects construction in urban settlements always exceed the available financial resources for a given plan period. Second important criterion is urban politics: it is desirable to invest in each neighbourhood that represents a defined territory, in order to achieve a certain balance for the users of public goods. Then, there are more technical, administrative and organizational conditions involved, all of which implies new criteria and limits, so that the model becomes more and more mathematically complex. That is why on this model the criteria addition is analyzed.

General mathematical model in the case of maximising the quantity in building infrastructural objects with limited financial resources has a standard form:

$$\max f(x) = \sum_j x_j;$$

With limits

$$\sum_j a_{ij} x_j \leq b_i,$$

$$x_j \geq 0, \ i \ \text{are whole numbers},$$

$$i =1,2,\ldots, m; j =1,2,3,\ldots,n,$$

Where: \(n\) - number of locations \(B_j\) where construction is planned, \(j \in J = \{1,2,\ldots,n\}\),

\(m\) - Number of infrastructural object types \(A_i\), \(i \in I = \{1,2,3,\ldots,m\}\),

\(a_{ij}\) - Financial resources needed for objects \(i\) on location \(j\),

\(b_i\) - Available resources for object \(i\),

\(x_j\) - Quantity in meters of objects which is going to be financed on location \(j\).
For financial resources to be rationally used, new problem optimising criteria is introduced, depending on given scopes of investment program, and these are:

- determining the disposition of available financial resources by location, so as to ensure construction of maximum physical quantity,
- another variant of that solution, when maximum building quantity is wanted only for certain locations,
- then optimisation of investment, if the condition is that on each location is built a minimum of work quantity, or
- if the condition is that on each location is maximised the possible building quantity.

**a. Maximisation of total building quantity on the full program level:**

Corresponding function to this additional criterion is:

\[(\text{opt}) f_j(x) = x_j, \text{ for } j, j \in J.\]

Introducing unknown values \(x_j\) for building quantities of \(j\) objects (measured in hundreds of meters) a concrete mathematical model is formed and it is the standard form, previously mentioned.

**b. Maximisation of building quantity in every settlement (location) in the program:**

This two-stage (bi-criterial) procedure requires defining firstly maximum work quantity by locations as a function of limits:

\[\max f_j^*(x_j) = x_j,\]

With limits

\[
\sum_j a_j x_j \leq b_i, \\
x_j \geq 0, \text{ } i \text{ are whole numbers,} \\
i=1, 2, \ldots, m; j=1,2,3,\ldots,n.
\]

Then the maximum program quantity is calculated using the function \(f(x)\) mentioned, so that the ideal value obtained from \(f_j^*\), is not disturbed, that is, the sequence for each location is two identical procedures.

**c. Maximisation of investment in only a few of programmed locations:**

When there is a request that for locations maximum resources are planned \(k\) \((k < j)\), it is necessary to resolve two models, with eventual variation of resources given \(S_{g_k} \leq \max f_{j^*}, J^*_k = \{k\}\).

Explanation is desired for general case: for total of four locations \((B_1, B_2, B_3, B_4)\) and three types of infrastructural objects \((A_1, A_2, A_3)\), if first two locations assigned maximum of resources, modelling is as follows:
Table 1:

<table>
<thead>
<tr>
<th></th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$b_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$</td>
<td>$a_{11}$</td>
<td>$a_{12}$</td>
<td></td>
<td></td>
<td>$b_1$</td>
</tr>
<tr>
<td>$A_2$</td>
<td>...</td>
<td>$a_{22}$</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$A_3$</td>
<td>...</td>
<td>$a_{33}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$a_{44}$</td>
</tr>
</tbody>
</table>

quantity $x_j$  $x_1$      ...  $x_3$  $x_4$  

First model:

$$\max f_1(x) = \sum_j x_j$$

$$\sum_j a_{ij} x_j \leq b_j, x_j \geq 0$$

Next model:

$$\max f(x) = \sum_j x_j$$

$$\sum_j a_{ij} x_j \leq b_j, x_j \geq 0, \max f_1(x) \geq Sg_k$$

d. Maximisation of total program quantity with a given minimal quantity in every location:

Limiting work quantity for each location on financial investments not less then $0 < \alpha < 1$ (i.e. $\alpha = 0.1$) out of total program quantity, new limits for previous general example are:

$$x_j \geq 0.1 \sum x_j, \text{ for every } j$$

$$B_1 \ldots x_1 \geq 0.1(x_1 + x_2 + x_3 + x_4) \quad \Rightarrow \quad 0.9x_1 - 0.1x_2 - 0.1x_3 - 0.1x_4 \geq 0$$

$$B_2 \ldots x_2 \geq 0.1(x_1 + x_2 + x_3 + x_4) \quad -0.1x_1 - 0.9x_2 - 0.1x_3 - 0.1x_4 \geq 0$$

$$B_3 \ldots x_3 \geq 0.1(x_1 + x_2 + x_3 + x_4) \quad -0.1x_1 - 0.1x_2 + 0.9x_3 - 0.1x_4 \geq 0$$

$$B_4 \ldots x_4 \geq 0.1(x_1 + x_2 + x_3 + x_4) \quad -0.1x_1 - 0.1x_2 - 0.1x_3 - 0.9x_4 \geq 0$$

Starting model is expanded by additional limits, as requested by investor.

e. Maximisation of total program quantity with a given maximal quantity in every location:

Analogously it is possible to have requests from investors management that to any given location is assigned the maximum of coefficient $\beta$ ( $0 < \beta < 1$) out of total available resources. New limits expand the basic model as follows:

$$x_j \leq \beta \sum x_j, \text{ for every } j$$
Practical work on optimisation is rationalised using synthesis procedure, as shown in the following charts (S - resources given):

\[
B_1 \ldots x_i \leq \beta (x_1 + x_2 + x_3 + x_4) \Rightarrow (1 - \beta)x_i - \beta x_2 - \beta x_3 - \beta x_4 \leq 0 \\
B_2 \ldots x_2 \leq \beta (x_1 + x_2 + x_3 + x_4) \Rightarrow -\beta x_1 (1 - \beta)x_2 - \beta x_3 - \beta x_4 \leq 0 \\
B_3 \ldots x_3 \leq \beta (x_1 + x_2 + x_3 + x_4) \Rightarrow -\beta x_1 - \beta x_2 (1 - \beta)x_3 - \beta x_4 \leq 0 \\
B_4 \ldots x_4 \leq \beta (x_1 + x_2 + x_3 + x_4) \Rightarrow -\beta x_1 - \beta x_2 - \beta x_3 (1 - \beta)x_4 \leq 0
\]

<table>
<thead>
<tr>
<th></th>
<th>a. (max f)</th>
<th>b. (max f)</th>
<th>c. (max f)</th>
<th>d. (max f)</th>
<th>e. (max f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B_1</td>
<td>(x_i = \ldots)</td>
<td>(S_j = \ldots)</td>
<td>(\sum S_j = \ldots)</td>
<td>(x_i = \ldots)</td>
<td>(\sum x_j = \ldots)</td>
</tr>
<tr>
<td>B_2</td>
<td>(\sum x_j = \ldots)</td>
<td>(\sum S_j = \ldots)</td>
<td>(\sum S_j = \ldots)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B_3</td>
<td>(\sum x_j = \ldots)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B_4</td>
<td>(\sum x_j = \ldots)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Investment efficiency index for simulated solutions

<table>
<thead>
<tr>
<th>variations</th>
<th>Investment plan (resources)</th>
<th>Planned building quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{max } f)</td>
<td>B_1</td>
<td>B_2</td>
</tr>
<tr>
<td>a.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Survey of all solutions
Conclusion

Contemporary computer techniques enable wide application of these models for concrete planning and monitoring of infrastructure building realisation in urban environments. The model provides essential help to the public companies experts in planning annual and plurals-annual programs of public building in cities and settlements.

REFERENCES